Barem de corectare şi notare OLM clasa a XI a M1

1 Demonstrația se face prin calcule directe sau folosind relația C-H şi se obține egalitatea cerută……………………………………………………………….3p

b)
$$A^2 = A\cdot A = \begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -6 & 4 \end{pmatrix} = -A$$
$$A^3 = A^2\cdot A = \begin{pmatrix} -3 & 2 \\ -6 & 4 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix} = A$$
…………………..1p.

rezulta că $A = A,\quad A^2 = -A,\quad A^n = \begin{cases} A, \text{ dacă } n = 2k-1 \\ -A, \text{ dacă } n = 2k \end{cases}$ …………..2p

demonstrație prin inducție matematică…………………………… …..1p

2. $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & a & b \\ 1 & a & 0 & c \\ 1 & b & c & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & -b & a-c & b \\ 0 & a-b & -c & c \\ 1 & b & c & 0 \end{vmatrix} =$ …………………………… …..1p

$$= -\begin{vmatrix} 1 & 1 & 1 \\ -b & a-c & b \\ a-b & -c & c \end{vmatrix} = -\begin{vmatrix} a-c+b & 2b \\ b-a-c & c-a+b \end{vmatrix} = a^2+b^2+c^2-2ab-2ac-2bc$$
……………………………………………………………………………2p

$$\begin{vmatrix} bc & bc-1 & -bc-1 \\ ac & ca+1 & -ac \\ ab & ab & 1-ab \end{vmatrix} = (ab+ac+bc)\begin{vmatrix} 1 & 1 & -1 \\ ac & ac+1 & -ac \\ ab & ab & 1-ab \end{vmatrix} =$$ ……………….2p

$$=(ab+ac+bc)\begin{vmatrix} 0 & 0 & -1 \\ 0 & 1 & -ac \\ 1 & 1 & 1-ab \end{vmatrix} = (ab+ac+bc)\cdot 1 = ab+ac+bc$$ ……………1p

Rezultă $a^2+b^2+c^2-2ab-2ac-2bc\ ab+ac+bc = a^2+b^2+c^2-ab-ac-bc =$

$$=\frac{1}{2}\left[(a-b)^2+(a-c)^2+(b-c)^2\right]\ge 0$$ ………………………………………1p

3. Avem cazul de nedeterminare 1^{∞} ……………………………………1p

După transformările necesare avem

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{b}+a-1}{a}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{b}+a-1}{a}-1\right)^{n}=\text{.......................................}2p$$

$$\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{b}+a-1-a}{a}\right)^{n}=\text{..}2p$$

$$\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{b}-1}{a}\right)^{n}=e^{\lim\frac{b^{\frac{1}{n}}-1}{\frac{1}{n}}\cdot\frac{1}{a}}\quad\text{...}1p$$

$$=e^{\ln b\cdot\frac{1}{a}}=b^{\frac{1}{a}}\text{...}1p$$

4. Deoarece $\lim_{n\to\infty}\frac{\ln n}{n}=0$ …………………………………………………2p

$$\lim_{n\to\infty}\frac{n-\ln n}{\ln(n-\ln n)}=\lim_{n\to\infty}\frac{n\left(1-\frac{\ln n}{n}\right)}{\ln n\left(1+\frac{\ln\left(1-\frac{\ln n}{n}\right)}{\ln n}\right)}=\lim_{n\to\infty}\frac{n}{\ln n}=\infty\quad\text{……………}3p$$

rezultă că limita cerută este -∞ …………………………………………2p